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HOW PEOPLE INTERACT  
Toward a General Theory of Externalities

by

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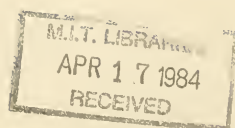
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## A B S T R A C T

This paper treats reciprocal externalities as a special case of how people interact. Two-person two-choice games are used to construct a theory and taxonomy of interactions and to show that separability of objective functions (utility, cost, or profit) is sufficient, but not necessary, for the existence of dominant strategies. The model is generalized to  $N \times 2$  games and applied to tax evasion, rural development, optimal saving and social norms.

## A C K N O W L E D G E M E N T S

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## HOW PEOPLE INTERACT

### Introduction

Economics is the study of relationships among people, which are expressed as relations among things, such as money, assets and goods. Three conclusions follow immediately from this unconventional definition. First, other disciplines, too, study relations among people -- sociology, anthropology, and psychology -- and economics can profit from their wisdom.<sup>1</sup> Second, those same behavioral and social sciences can profit from the rigorous, abstract way economics models behavior.<sup>2</sup> Third, one of economics' most powerful tools for modelling human relationships directly is game theory. Economics shares with many other disciplines a deep and abiding curiosity about how people get along with one another (or fail to). An attempt to construct an economic theory of how people interact -- one applicable to other disciplines -- should find game theory a terse and pliable language whose realm stretches far beyond the economic province of markets.<sup>3</sup> This paper comprises such an attempt.

At times, economics seems split into two disparate subdisciplines: microeconomics, study of how individuals behave, and macroeconomics, study of how the national economy behaves; the gap between them is both broad and deep. Yet, there is at least one common theme that unites the two. It is the injustice, instability and waste that results in human society from the absence of certain markets. Nobel Laureates Kenneth Arrow, a microeconomist, and James Tobin, a macroeconomist, each referred to this issue in their Stockholm addresses.

Early in their speeches, they each stated a position that can be merged and paraphrased:

If a complete set of perfectly competitive markets exist for all goods, services and assets exist (including future and contingent markets), and there is perfect costless information available to all, then prices --relations among things -- bring both microeconomic and macroeconomic relations among people into harmony. (Arrow, 1975; Tobin, 1982).

There is thus broad agreement that a full set of ideal markets is sufficient to eliminate inefficiency, involuntary unemployment and the business cycle.<sup>4</sup>

The Arrow-Tobin maxim, then, implies that every case of market failure -- failure of existing free markets to maximize social welfare -- can be attributed to some market imperfection, or missing market. Moreover, every instance of externalities, or external costs and benefits -- defined as the effects of one person's actions on other people's wellbeing, expressed in any way except through prices (Varian, 1978) -- can also be attributed to missing markets (Coase, 1960).<sup>5</sup> The central issue of modern economics, therefore, turns out to be the nature, causes and remedies of missing markets.

It is an issue of great interest to other disciplines. Biology (tragedy of the commons), Law (liability rules), regional science (spillover effects) and political science (free rider phenomena) all have their own special brands of externality problems. In social choice theory, externalities are a necessary condition for cyclical-- that is, problematic, in Arrow's sense -- social preferences (Bernholz, 1982).

How should one best model missing markets? The dominant approach is Walrasian general equilibrium. There are at least two difficulties with this approach. One is that general equilibrium modelling must assume the nature of the social institutions in which markets operate. This begs the question. Often, we seek to deduce what social institutions could remedy imperfect markets. In this task, Walras is not helpful. A second difficulty is that modelling markets that do not currently exist requires a breadth of imagination economists rarely possess -- with a few notable exceptions (Stein, 1971; Colander & Lerner, 1980).<sup>6</sup> Game theory obviates both of these shortcomings. It enables us to infer the type of social institutions likely to arise from a given game setting (Schotter, 1980; Ulmann-Margalit, 1978).<sup>7</sup> And by mapping directly from actions into utility, game theory emphasizes the heart of the problem -- relations among people -- while letting us skip the hard task of modelling nonexistent goods, markets or technologies.<sup>8</sup> Further, when macroeconomic conflict is structured as a game -- generally, non-zero-sum -- the behavioral, micro underpinnings are made explicit, and progress is made in reuniting micro and macro theory.<sup>9</sup>

A useful distinction may be drawn between economic exchange and social exchange. In economic exchange, goods, services or assets change hands at clearly specified prices. When others are affected by the transaction -- other than by its impact on market price itself -- there are externalities generated. In social exchange, obligations are not clearly specified in advance. An action may induce a response, but it is usually less clear-cut and more diffuse than when buyer and seller agree on a mutually acceptable price.

Every action, therefore, may be characterized by two key parameters: the number of other people affected by it, directly or indirectly, and the degree of certainty attached to others' response. Figure 1 maps actions according to these two parameters. A large class of economic behaviors involves only the doer, when, for instance, individuals decide how much of their income to spend and how much to save, or how much of their time to devote to work and how much to leisure. The opportunity costs of consumption of goods and leisure are known and certain, but there may be external costs or benefits when others are affected. In market transactions, with buyers and sellers involved, economic exchange may also engender externalities. In a broad sense, externalities are special cases of social interaction arising from social exchange. A model of how people interact will therefore include externalities as a special case. Game theory is especially suited for modelling social exchange; an individual's behavior (choice of strategy) is usually built on an expectation of what the opponent will do,

Number of Other  
People Affected

M a n y

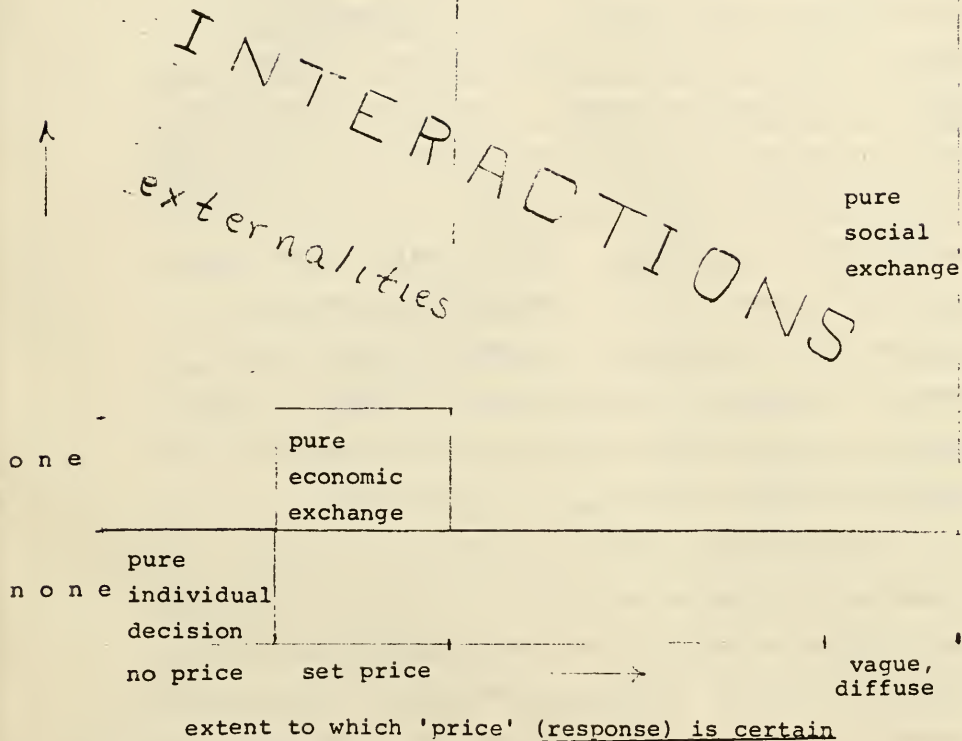


Figure 1. Externalities, Interactions and  
Economic and Social Exchange



but that response is rarely known with certainty. By treating interaction as a kind of game, both simplicity and generality are attained. One interpretation of a game is that it reflects the workings of an informal type of "interactions" market. One player chooses his or her strategy in the knowledge that it will affect other people, who may, in turn, respond in a manner favorable or unfavorable to the player. What is being traded are gains and losses -- without contractual prices, and without certain consequence attached to each player's choice of strategy. The vagueness of "prices" in such a market is a disadvantage outweighed by its emphasis on how people behave and interact, in a direct manner.

Treating externalities as a game also helps to highlight their reciprocal nature. As Calabresi has noted, Taney's pollution may impose an external cost on Marshall, but it is equally true that Marshall's injunction preventing Taney's pollution imposes an external cost on Toney. In a tautological sense, all externalities are reciprocal, a fact, Calabresi notes, that the courts at times fail to understand.  
//



The plan of the paper is as follows. Interactions are first portrayed as  $2 \times 2$  noncooperative games. Twelve basic interaction types are identified, and are shown to generate the 78 possible conceptually-different  $2 \times 2$  games. Oblique utility functions -- which map directly from strategy choice into utility -- are used to draw a distinction between linear and nonlinear interactions. It is shown that nonlinear interactions, of a certain type, can generate games with dominant strategies, contrary to an implication in Davis & Winston (1962) and Bacharach (1979). An example is given where two firms' interrelated, nonlinear production functions still generate a game with a dominant strategy. An inventory of questions that  $2 \times 2$  game matrices can help answer is outlined, and is seen to be broader than the existence-uniqueness-stability criteria of Walrasian general equilibrium. Several applications of the  $2 \times 2$  taxonomy are given.

The model is then generalized to  $N \times 2$  games (that is,  $N$  players, each of whom chooses one of two strategies). Each of 12 basic  $N$ -person interaction types is graphed. The model is then applied to tax evasion, optimal saving, rural development and social norms.

## Interactions as 2x2 Games

Consider the following parable. There are two persons, or two groups. Each faces a strictly dichotomous choice between two types of behavior: passive, which has no effect on either the player or his or her opponent; and active, which affects both player and opponent, either for good or for ill. The active behavior therefore generates costs or benefits external to the person doing it. There are four possible outcomes to this game; the final outcome is determined only after both players act, since neither knows with certainty the other person's choice. The game may be one-time or repeated.

Assume that each individual's preferences are strongly ordered, so that the four game outcomes are ranked by each player with strict inequalities. There are  $4! = 24$  ways to order four outcomes. However, only 12 of these orderings are unique. By renaming the strategy choices -- for instance, call the externality-generating choice, "not smoking", instead of "smoking" -- rows and matrices of the 2x2 game matrix are interchanged, yet the game is no different.

Let the Row player's strategy be  $X (= x, x')$  and the Column player's strategy be  $Y (= y, y')$ . A set of 12 interaction types for Row player is shown in Figure 1. These will become the building blocks of a taxonomy, or typology, of externalities.

Suppose Column player has the same preference orderings as Row. Then the game is symmetrical, and there are 12 such games. Now, suppose Row and Column have different preference orderings.

	y	y'
x'	2	3
x	1	4

#1

3	2
4	1

#2

2	4
1	3

#3

4	2
3	1

#4

3	4
1	2

#5

4	3
2	1

#6

1	4
3	2

#7

4	1
2	3

#8

2	4
3	1

#9

4	2
1	3

#10

3	4
2	1

#11

4	3
1	2

#12

Figure 2. Twelve interaction types

Then, there are  ${}_2C_{12} = 12!/10!2! = 66$  different games. That makes a total of 78 conceptually-different 2x2 games, as Rapaport & Guyer (1966) showed. They comprise an exhaustive catalog of an admittedly limited class of externalities, dichotomous ones.

### Linear & Nonlinear Interactions

At this point, it is helpful to ask: what sort of utility function is capable of generating all 12 interaction types in Fig. 2? And what may be learned from this function?

The type of utility function with which we shall work differ from conventional ones, in that they map directly from behavior into utility, rather than from commodities. These are known as oblique utility functions, as opposed to direct or indirect ones.

*An (ordinal) oblique utility function is a direct mapping from a player's strategy choice, and those of other players, into that player's utility:*

$$U = U(X,Y)$$

where  $U$  is the Row player's utility,  $X$  is the Row player's strategy, and  $Y$  is the Column player's strategy. For simplicity, we shall remain in the context of two-person, two-choice games.

Oblique utility functions are useful for drawing an important distinction between two types of interpersonal relations: Linear (or separable) and non-linear (nonseparable).

*A Linear interaction (L) is one where (a) a player's total utility  $U( )$  is a function of other players' behavior, but (b) the marginal utility of a player's strategy choice is independent of what other players choose to do.*

The simplest representation of a linear interaction, for 2x2 games, is:

$$U = aX + bY \qquad a, b \geq 0 \\ X, Y = 0, 1$$

*A Non-linear interaction (NL) is one where both (a) the marginal utility of a player's decision choice is dependent on opponents' behavior, and (b) the marginal utility of opponents' strategy to the player depends on the player's own strategy choice.*

The simplest functional form that expresses these two conditions is:

$$\frac{\Delta U}{\Delta X} = a + c Y$$

$$\frac{\Delta U}{\Delta Y} = b + d X$$

(Because X and Y are discrete, the finite difference operator  $\Delta$  is used).

Apply the finite integration operator  $E = \Delta^{-1}$  to the above two equations (Richardson, 1954) gives:

$$U = K + a X + c X Y$$

K, J = constants of integration

$$U = J + b Y + d X Y$$

These two equalities are satisfied when  $K = bY$ ,  $J = aX$ ,  $c=d$  :

$$U = a X + b Y + c X Y$$

This is the simplest possible functional form for non-linear interaction.

The dozen preference orderings shown above in Fig.2 serve as the building blocks of the theory of interactions portrayed here. It is worth asking: Which of those preference orderings are inherently linear in nature, and which are non-linear? That is, which of the 12 matrices can be generated by a utility function of the form:  $U = aX + bY$ , and which require a third interactive term,  $cXY$  ? This question turns out to have considerable importance in the economic theory of externalities.

*Preference types 3, 4, 5 and 6 are linear.  
Preference types 1, 2, 7, 8, 9, 10, 11 and 12  
are non-linear.*

When both players have preference type 3 (i.e. preferences are "symmetrical"), the game is Prisoner's Dilemma. This well-known game is the only 2x2 game with a single stable Pareto-inferior equilibrium. When the Column player's utility function is characterized as  $V = rX + sY$ , Prisoner's Dilemma results when:

$$a > 0, \quad b < 0, \quad a < |b|$$

$$s > 0, \quad r < 0, \quad s < |r|$$

In the economic literature on externalities, it is sometimes implied that the fundamental problem

is NL, non-linear interdependence:

The 'twisted' form of the interdependence of the two firms not only robs us of a clear solution, but it also, as Davis and Whinston have argued... makes the policy problem intractable. At least, the classical Marshall-Meade taxes on the two outputs which worked in the separable case do not work now. These taxes can still shift the equilibrium pair [of strategies] to a socially better position; but they cannot make that pair dominant. (Bacharach, 1977).

But, as Davis & Whinston (1962) explicitly note, in the case of Prisoner's Dilemma, individually optimal behavior may be collectively disastrous. Here, the problem is not that marginal utilities (costs, profits, or whatever) are interdependent, hence uncertain and likely to lead to non-optimal choices; rather, marginal utilities are constant, but are such that what one player gains from some behavior  $X=1$  is much smaller than the loss that behavior imposes on the opponent, and when all players seek such gains, all of them lose. (Davis & Whinston suggest that two firms playing Prisoner's Dilemma solve the problem by merging; under anti-trust constraints this is not always feasible, nor is it an admissible solution for individuals, unless perhaps they marry).

If it is true that linear interactions are problematic, it is equally true that non-linear interactions need not be.



In Davis & Whinston (1962), the claim is made several times -- though not stated as a theorem -- that if externality interactions are non-linear, then the game lacks an equilibrium, because no player has a dominant strategy. For instance:

From the game-theoretic standpoint, this type of [nonseparable, i.e. nonlinear] externality suggests the absence of dominance. ...the optimal output strategy of one firm depends upon the output strategy selected by the other firm. Such interdependence is the essence of non-dominance. (p. 254)

"...from the standpoint of discrete games, the presence of non-separable externalities suggests there is no row-column dominance". (p. 255)

"There seems to be no a priori method for determining what strategies firms will select in the presence of non-separable externalities".

More recent literature has reaffirmed this position:

...the solution of the non-separable externalities problem, like that of the duopoly problem, has not been determined by our theories. Game theory has thrown light on the problem, but it has not provided a definitive answer. (Bacharach, 1977, p. 77)

The definitive answer game theory provides is this:

*Preference types 11 and 12 are non-linear, yet generate symmetrical games with clear-cut Nash equilibria, i.e. dominant strategies for both Row and Column players.*

The 2x2 symmetrical games generated by preference types 11 and 12 are:

3	4
2	1

4	3
1	2

I shall now prove that preference ordering 11 is non-linear, i.e. cannot be generated by a linear utility function. The proof for preference type 12 is the same. Since both preference types generate dominant strategies, it follows that linearity (or separability) is NOT a necessary condition for the existence of dominant strategies, (though of course it is a sufficient condition).

Consider preference type 11:

	$Y=y'$	$Y=y$
$X=x'$	3	4
$X=x$	2	1

Here, Row clearly has a dominant strategy:  $X=x'$ . Yet it is easy to show that the oblique utility function underlying the above preference type is necessarily nonlinear, i.e. cannot be written as  $U = aX + bY$ .

Assume the contrary, i.e., there exists some  $U = f(X) + g(Y)$ ,  $X = x, x'$ ;  $Y = y, y'$ . Then, according to the above matrix:

$$U(x', y) > U(x', y') > U(x, y') > U(x, y)$$

If  $U(\ )$  is separable, then these equalities may be written:

$$f(x') + g(y) > f(x') + g(y') > f(x) + g(y') > f(x) + g(y)$$

The first two inequalities imply:

$$g(y) > g(y')$$

and the last two inequalities imply:

$$g(y') > g(y)$$

This is a contradiction; hence,  $U$  is not separable.

An identical proof applies to preference type 12.

An Example from Meade (1952)

James Meade's classic (1952) article on externalities discusses the "Atmosphere Case", with interacting production functions:

$$x_1 = H_1(L_1, C_1) A_1(x_2)$$

$$x_2 = H_2(L_2, C_2) A_2(x_1)$$

where  $x_i$  is output of firm  $i$ ,  $L$  and  $C$  are labor and capital inputs, and  $A( )$  shows the effect of one firm's production decision on the other's output (e.g., by pollution). Citing this example, Davis & Whinston argue that "this type of interdependence [i.e. nonseparable] creates uncertainties which...make such an assumption [existence of a corrective tax-subsidy scheme] arbitrary and unwarranted. ...In the non-dominance case a stable equilibrium is unlikely to be achieved." (p. 257). Here is a counterexample, where nonseparability (that is, nonlinearity) is preserved, but where each firm has a dominant strategy, and hence a stable equilibrium exists.

Let:

$$x_1 = L_1(1 + Ax_2) + BL_2 \quad A, B = \text{constants}$$

$$x_2 = L_2(1 + Ax_1) + BL_1 \quad L_1, L_2 = 0, 1$$

Suppose each firm chooses a dichotomous labor input  $L = 0, 1$ .

Let  $A = -1/2$  and  $B = 1/4$ . This is a 2x2 game, whose matrix is:

	$L_2=1$	$L_2=0$
$L_1=1$	5/6 , 5/6	7/8 , 1/4
$L_1=0$	1/4 , 7/8	0 , 0

Both firms have a dominant strategy:  $L_1=L_2=1$ . This equilibrium is strongly stable. This game is the equivalent of Game #11. The above configuration of payoffs can never be generated by separable functions.

To summarize: the following result seems to me to have been implicit in the literature on externalities since the classic paper of Davis & Whinston (1962):

separable  $\longrightarrow$  dominant strategy,  $\longrightarrow$  existence of  
 preferences  $\longleftarrow$  stable equilibrium  $\longleftarrow$  tax/subsidy

I contend that the following small but not unimportant amendment must be made:

separable  $\longleftarrow \text{ // } \longrightarrow$  dominant  $\longleftarrow$  existence of  
 preferences  $\longrightarrow$  strategy  $\longrightarrow$  tax/subsidy

More precisely:

nonseparable  
 preferences 11, 12  $\longrightarrow$  dominant  $\longrightarrow$  existence of  
 OR  $\longleftarrow$  strategy  $\longleftarrow$  tax, subsidy  
 separable  
 preferences

### Categories of Externality Games

Table 1 classifies the 78 possible interaction types, or externality games, according to whether the (simplest possible) utility function that can generate their underlying preferences is linear or nonlinear. While the table tells us nothing about how actual externality games are distributed, it does imply that the potential number of nonlinear interactions -- interactions where at least one preference set is nonlinear -- is far greater than the potential varieties of purely linear interactions. Social psychologists, for instance, may wish to subscribe to what econometricians have known for many years: that the world is nonlinear, in its preferences as much as in its technology.

Nature of Players' Utility Functions				
P r e f e r e n c e s		Both are Linear in X & Y	One of Two is Non- Linear in X & Y	Both Are Non-Linear in X & Y
	Same for Both Players (Symmetric)	4	-	8
	Different for Each Player (Asymmetric)	6	3 2	2 8
	Total	1 0	3 2	3 6

Table 1. 2x2 Externality Games, Classified by 'L' and 'NL' Utility Functions and Symmetry

*Each cell shows the number of externality-game types, out of a total of 78 possible ones, according to whether players' utility functions are linear or non-linear in X and Y, and whether players preference orderings are the same, or different. Utilities that are linear in X and Y can generate only 10 of the 78 possible externality-games.*

# The Case of Weak Preferences

So far, I have assumed strongly ordered preferences, i.e., no ties among the four outcomes of  $2 \times 2$  games. Guyer and Hamburger (1968) showed that for weakly-ordered preferences, 732 conceptually different  $2 \times 2$  games are possible. It can be shown that the nonlinear utility function:  $U = aX + bY + cXY$  can generate, for appropriate values of  $a$ ,  $b$  and  $c$ , all possible permutations of tied outcomes (no ties, two ties, three ties, and four ties), with  $c=0$  (linear interdependence) as a special case, and hence can generate all 732  $2 \times 2$  games.

a) No ties: discussed above, for strongly-ordered preferences

b) Two outcomes are tied: There are  $4!/2!2! = 6$  ways that two outcomes can be tied. They are:  $b=0$ , implying that  $a=-c \neq 0$ ;  $a=0$ , implying  $b=-c \neq 0$ ;  $a=b$ , implying  $c \neq -a \neq 0$ ;  $b=a+b+c$ , implying  $a=-c \neq 0$ ;  $a=a+b+c$ , implying  $b=-c \neq 0$ ; and  $0=a+b+c$ , implying  $c \neq -b \neq -a \neq 0$ .

c) Three outcomes are tied: There are  $4!/3!1! = 4$  ways this can happen: They are:  $0=b=a+b+c$ , implying  $a=-c \neq 0$ ;  $0=a=a+b+c$ , implying  $b=-c \neq 0$ ;  $0=a=b$ , implying  $c \neq 0$ ; and  $a=b=a+b+c$ , implying  $a+b=-c \neq 0$ .

d) Four outcomes are tied: This can only happen when  $0=a=b=a+b+c$ , implying  $a=b=c=0$ .

(The following matrix may be helpful in working through the above permutations:

	Y=1	Y=0
X=1	a+b+c	a
X=0	b	0



Structuring interactions in the form of  $2 \times 2$  games (later,  $N \times 2$ ) has important advantages. Once the structure of the game matrix is known, a great many questions about the nature, stability and compatibility of interactions can be answered. Moreover, we can learn much about the likely outcome of the game itself, and whether this outcome would benefit by some form of outside intervention. One of the attractions of game theory is that, unlike conventional general equilibrium modelling, it need not confine its attention solely to characterizing equilibria. The simple act of constructing a game matrix forces us to think about non-equilibrium outcomes. These outcomes may become important when the dynamics of the game unfold, or when players err, or misjudge their opponents, when no equilibrium exists, or when players' motives are more complex than individual utility maximization.

What follows is an inventory of 10 questions that illustrates how one might go about dissecting an externality game. The concepts of "threat-vulnerable" and "force-vulnerable", and competitive pressure, are due to Rapaport & Guyer (1966); the concept of "nonmyopic equilibrium" and absorbing outcomes are due to Brams and Wittman (1980), and Brams and Hessel (1982), respectively.<sup>12</sup> See also Hirshleifer (1983). For an application of the inventory to family therapy, see Maital and Maital (1983b).

1. Do both players have a dominant strategy?

If Yes: The resulting outcome is an equilibrium.

2. Does only one player have a dominant strategy?

If Yes: The equilibrium is defined by the opponent's best response to the dominant strategy.

3. Does neither player have a dominant strategy?

If Yes: There may be zero, or two equilibria.

4. Is the equilibrium Pareto-efficient?

If Yes: Every change in strategy makes at least one player worse off.

5. Does the largest payoff to both players occur in the same cell (outcome)?

If Yes: The game is one of no conflict.

6. Can one player achieve a better relative payoff (compared to the opponent), by a change in strategy, even though his absolute payoff is smaller?

If Yes: There is "competitive pressure, or rivalry" in the game.

7. Can one player, by threatening to change his strategy, cause the other player to change his strategy?

If Yes: The game's outcome is "threat vulnerable".

8. Can one player, by actually changing his strategy, force the other player to change his?

If Yes: The game is "force vulnerable".

9. In sequential games, if equilibrium exists, is it "nonmyopic", (i.e. both players would end up worse off if they departed from it)?

10. Where nonmyopic equilibrium does not exist, are there "absorbing outcomes" (i.e. one or two outcomes to which departures from worst and next-to-worst outcomes converge) ?

## Applications

### A. Blessed Are the Givers

In the famous William Sydney Porter (O. Henry) story, The Gift of the Magi, Jim and Della each want to buy the other a special Christmas present as a surprise. Della has only \$1.87, 60¢ of it in pennies; Jim is flat broke. Della sells her long hair for \$20 (unbeknown to Jim) to buy a new chain for Jim's proudest possession, a watch. Jim sells his watch to buy Della fancy combs for her long hair, without telling Della. They exchange presents on Christmas, with paradoxical result.

Suppose both Jim's and Della's preferences are as follows: "It is more blessed to give than receive" (Acts: 20, 35); both giving is bad, but neither giving is worst of all. Let each choose independently between giving a gift (financed by sacrificing a prized possession) or not giving. The game matrix for each is shown in Figure 3, the first entry in each cell is Jim's "utils," the second entry, Della's. (We here define "giving" as the externality-generating action:  $X = 1$  and  $Y = 1$ .)

Neither Jim nor Della has a dominant strategy. The initial position, or pseudo-equilibrium, is therefore likely to be the 'maximin' strategy

# DELIA

Give:  $Y=1$     Don't:  $Y=0$

J I M	Give $X=1$	2, 2	4, 3
	Don't $X=0$	3, 4	1, 1

Fig. 3

# DELIA

Give:  $Y=1$     Don't:  $Y=0$

J I M	Give $X=1$	2, 2	3, 4
	Don't $X=0$	4, 3	1, 1

Fig. 4

for each, defined as the strategy that maximizes the minimum gain. For each this is "giving" ( $X = 1, Y = 1$ ). This outcome, however, is Pareto-inferior. It can be improved by having one, and only one player concede. The first player to concede (shift to "don't give") gets the second-best outcome, but the player who does not concede and holds fast to "give" gets the first-best outcome. This game is sometimes called "Hero." It is a game of forbearance, where the player who holds out longest eventually comes out on top. When both hold out, however, both are worse off. Both Della and Jim have interaction type #9. The particular externalities game they play can be modeled as:  $U = 4X + 2Y - 5XY$ ,  $V = 2X + 4Y - 5XY$ , Jim = U, Della = V.

Now, suppose Jim and Della are more egotistical, and think it is nice to give but nicer to receive.

The game model is, for example:  $U = 3X + 4Y - 5XY$ ,  $V = 4X + 3Y - 5XY$ .

The appropriate game matrix is Figure 4. This game is rather different.

The initial position remains  $X = 1, Y = 1$ , the maximin strategies for both Jim and Della. Now, however, the game switches from one of forbearance to one of preemption. Whoever concedes first and lets the other give a

present gets his or her first-best outcome; whoever lags gets only his or her second-best outcome. As in the Wild West, when both are equally quick on the draw, the result is mutual annihilation ( $X = 0, Y = 0$ ). Such games are often played by duopolies. (Suppose two firms each consider investing large sums to develop a new product, where the market is large enough only to support one producer. In this race, the first to enter the market is likely to win. When both enter, both lose. Uncertainty about the other firm's intentions may keep each from developing the product.)

B. Keep on Truckin'

Persons A and B jointly own a semi-trailer, and have a contract to transport oranges from Florida to New York. At times, only one of them does the haul, while the other rests at home; other times, they both do the haul and share the driving. A loves trucking, and gets more pleasure from driving than from staying home and letting B drive. A also dislikes B somewhat, so that having B along with him detracts from the pleasure he gets from his work. In contrast, B hates most of all driving alone, and greatly enjoys puttering at home in his garden, but likes best of all driving with company--even if it is A. Let each face a choice between drive ( $X = 1$ ),

$Y = 1$ ) and not drive ( $X = 0, Y = 0$ ).

A utility function consistent with B's preferences is:  $V = X - Y + 2XY$ .

This is interaction type #1. A's utility function might be:  $U = 4X + Y - 2XY$ .

This is interaction type ###. The externality game matrix is shown in Figure 5.

How will this game turn out? What is each player likely to do?

Of the two, only A has a dominant strategy: drive ( $X = 1$ ). Knowing this, B too will choose to drive. The result is Pareto-efficient. It may, however, turn out to be unstable. A has at least two strategic options for getting B to stay at home, thus giving A his best outcome, rather than second best, and moving B from his best to second best. These options are:

(1) threat: A can threaten not to drive ( $X = 0$ ) unless B stays home.

This would, of course, cost A heavily in utility, but it would cost B even more, since B would then get his worst outcome: driving alone. If this threat is credible, B will prefer to stay at home rather than drive alone. A's threat to switch from  $X = 1$  to  $X = 0$  may be sufficient to get B to switch from  $Y = 1$  to  $Y = 0$ .

(2) force: Suppose A's threat doesn't work. B finds it not credible.

A may then use "force" by actually shifting from  $X = 1$  to  $X = 0$ . B will

		TRUCKER P:	
		<i>Drive</i> <i>Y=1</i>	<i>Don't Drive</i> <i>Y=0</i>
TRUCKER A	<i>Drive</i> <i>X=1</i>	3, 4	4, 3
	<i>Don't Drive</i> <i>X=0</i>	2, 1	1, 2

Fig. 5



then find it worthwhile to shift, too, from  $Y = 1$  to  $Y = 0$  (not drive).

Once B stays at home, A shifts again, and starts driving alone. This outcome is, however, unstable, because as soon as A starts to drive, B joins him. The process may thus cycle endlessly.

There is a third process which makes the outcome:  $X = 1, Y = 1$  unstable. This is competitive rivalry. Suppose A is upset that B gets his best outcome, while he, A, gets only his second best. By refusing to drive ( $X = 0$ ), A can inflict more damage on B than he incurs himself. If A is more interested in maximizing the gap between his own outcome and his partner's than in attaining the best absolute outcome, A will shift to  $X = 0$  and try to make B drive. B may then respond by shifting to  $Y=0$  (don't drive). This forces A to shift back to  $X=1$  (drive), whereupon B will now choose not to drive ... and so on.  $X=1, Y=1$  is a Nash equilibrium, but behaviorally, it may be a highly unstable one.

# Generalization to N-persons

It is useful to generalize the above 2x2 games discussion to Nx2. That is easily done using a technique suggested simultaneously by Thomas Schelling (1973) and Henry Hamburger (1973). Retain the assumption of dichotomous choice, but assume now that there are N+1 players: the Row player, along with N opponents. Let Row's utility function be:

$$[1] \quad U = a(v) X + b(v), \quad X = 0, 1$$

where  $v$  is the number of other players whose choose  $Y = 0$ .

Note that this specification allows both linear [ $a(v) = \text{constant}$ ] and nonlinear [ $\Delta^2 U / \Delta X \Delta v \neq 0$ ] interaction. It lends itself to simple diagrams; for symmetric games (i.e. all N+1 players have the same preference orderings), graphing two curves,  $U_{X=0}$  and  $U_{X=1}$  as a function of  $v$  permits a complete static and dynamic analysis of the game:

$$[2] \quad \begin{aligned} U_{X=0} &= b(v) \\ U_{X=1} &= a(v) + b(v) \end{aligned}$$

Its interpretation is as follows --  $a(v)$  is Row's marginal utility from choosing  $X=1$  (instead of  $X=0$ ), and may be constant (linear interaction), or may depend on  $v$  in a simple or complex manner (nonlinear);  $b(v)$  determines the direct marginal utility (to Row) of one other player choosing  $Y=0$  (instead of  $Y=1$ ) and may be constant, or variable.

Two useful numbers may be derived from (2). First, in the event that there is no dominant strategy, i.e.  $U_{X=0}$  and  $U_{X=1}$  intersect at some  $v$ ,  $0 < v < N$ ; that value of  $v$  is interpreted as the smallest number of other players who choose  $Y=0$ , such that it becomes worthwhile for Row to switch strategies from  $X=0$  to  $X=1$  (or vice versa, as the case may be). It is found by solving:

$$[3] \quad U_{X=0} = b(v) = U_{X=1} = a(v) + b(v)$$

or,

$$a(v) = 0$$

Second, for games like Prisoner's Dilemma, where  $X=Y=1$  is the dominant strategy, it is interesting to calculate the size of the smallest coalition of those who play  $X=Y=0$ , such that Row's utility in such a coalition equals his or her utility when everyone plays  $X=Y=1$ : Solve the following for  $v$  :

$$[4] \quad a(0) + b(0) = b(v)$$

### Taxonomy & Graphs

Figure 6 repeats the taxonomy of Figure 2, but adds a graphic portrayal of each  $N \times 2$  game.  $U( )$  is Row's utility, and ' $v$ ' is the number of Column players who pick  $Y=y$ . Short parables are attached to each externality game

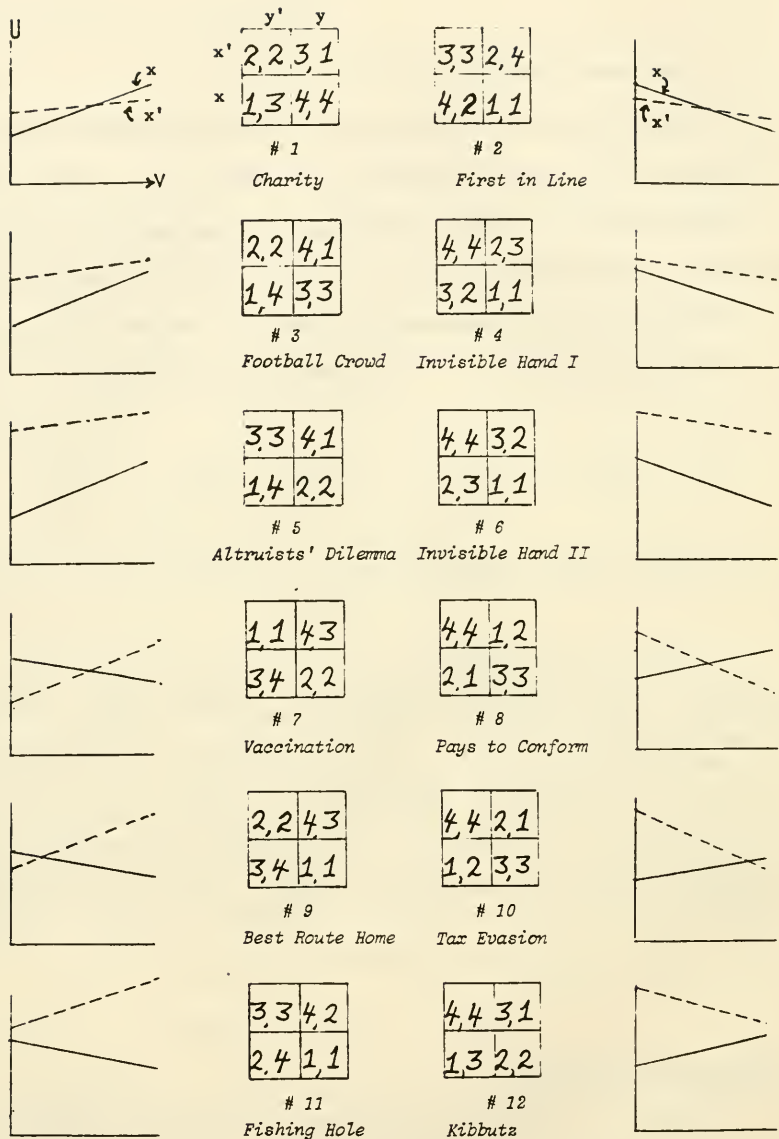


Figure 6. Two players or groups each must choose one of two strategies:  $x$ ,  $x'$ ; and  $y$ ,  $y'$ , respectively. When all players have identical preference orderings, 12 symmetric games emerge. Even-numbered games are the 'converse' of odd-numbered games; 'V' is the number of Column players who pick  $Y=y$ .

Note:  $v^*$  is the point at which  $x$  and  $x'$  intersect (if at all).

- #1 *Charity* :  $x$ =donate;  $x'$ =not donate. If I think  $v^*$  people will give, I will too; if I think less than  $v^*$  people will give, I refuse.  
The more others give, the better for me.
- #2 *First in Line*: A limited number of Super Bowl tickets are put on sale, first-come, first-served;  $x$ =rush to queue;  $x'$ =wait patiently. Rushing pays only if I am among the first. The equilibrium queue is  $v^*$ .
- #3 *Football Crowd*: In an exciting moment in a football game, I can either remain seated ( $x$ ) or stand ( $x'$ ); for me, it is best to stand, but when everybody acts this way, nobody can see; outcome is 3rd best
- #4 *Invisible Hand I*  $x'$ = try hard to accumulate wealth;  $x$ =renounce worldly goods.  
The more people are ascetic, the poorer society (and I) becomes.  
Everyone's dominant strategy is to amass wealth; outcome is 1st best
- #5 *Altruists' Dilemma*:  $x$ =help others;  $x'$ =help only yourself. Paradoxically, if I (for any reason) choose to be altruistic, and others do, too, people get in each others' way trying to be helpful: outcome "3".
- #6 *Invisible Hand II* As Invisible Hand I.
- #7 *Vaccination*  $x'$ = be vaccinated;  $x$  = refuse vaccination. The fewer other people are vaccinated, the more utility I gain from vaccination.  
Equilibrium is at  $v^*$ ; some, but not all, will choose  $x'$
- #8 *Pays to Conform*  $x$ =stop when light changes to amber;  $x'$ =don't stop. If most people stop, it pays to stop; if most people don't, it pays not to.  
Ultimately, everyone will conform to what becomes the social norm.
- #9 *Best Route Home*  $x'$ =take Route 1;  $x$ =take Route 2. Route 2 is fastest if nearly empty, but quickly clogs up. The trick is to pick the route others don't take; at equilibrium,  $v^*$  take Route 2.
- #10 *Tax Evasion*  $x$ =pay true tax;  $x'$ =evade. If almost everyone pays their taxes, enabling low rates, I will, too. If less than  $v^*$  are honest, the system collapses, and everybody evades.
- #11 *FishingHole* There are two fishing spots,  $V$  and  $W$ ;  $V$  dominates  $W$ ,  $x=W$ ,  $x'=V$ . The more other people choose  $W$ , the better  $V$  is for me.  
Everyone fishes at  $V$ , though some probably should be at  $W$ .
- #12 *Kibbutz*  $x$ =loaf;  $x'$ =work hard. I enjoy working hard best if everyone else does, but suffer if I work hard and others loaf. When I choose to loaf, the guiltier I feel, the more others work hard.

## Applications

### A. Tax Evasion

Let the dichotomous choice be between evading taxes entirely (X and Y equal one), thus placing a heavier burden on others (external cost), or paying them fully (X and Y are zero). Suppose that some individual's

marginal utility from evading taxes is constant, so that  $a(v) = \bar{a} = 10$ .

This will hold if evasion adds a constant sum to his income. Suppose

also that the marginal disutility to this individual caused by other

people evading taxes declines, as the number of honest people grows,

according to:  $b(v) = -20 + v$ . (Assume the interaction term is

zero). Let there be 20 people in society (N). Then,  $U = 10X - 20 + v$ ,  $X=0,1$  (See Fig. 7).

Our individual will be indifferent between having everyone evade taxes (including himself), and joining a coalition of 10 other people who vow honesty;  $v^* = 10$  is the solution to:

$$(14) \quad -10 = -20 + v \quad v = \text{number of those who do not evade}$$

This minyan is, of course, unstable. It pays any member of the coalition

to withdraw unilaterally, break the agreement and evade taxes, provided

at least some other people stay honest. When everybody evades, however,

crime doesn't pay. This is a description of a multiperson prisoner's

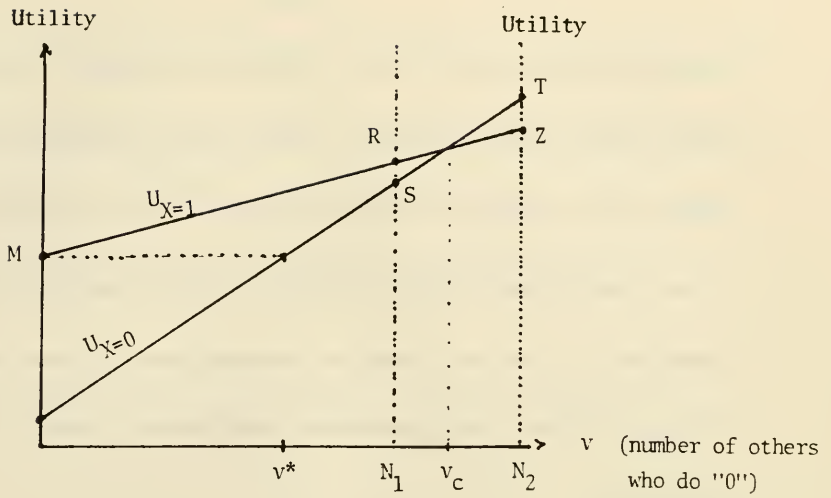


Figure 7.

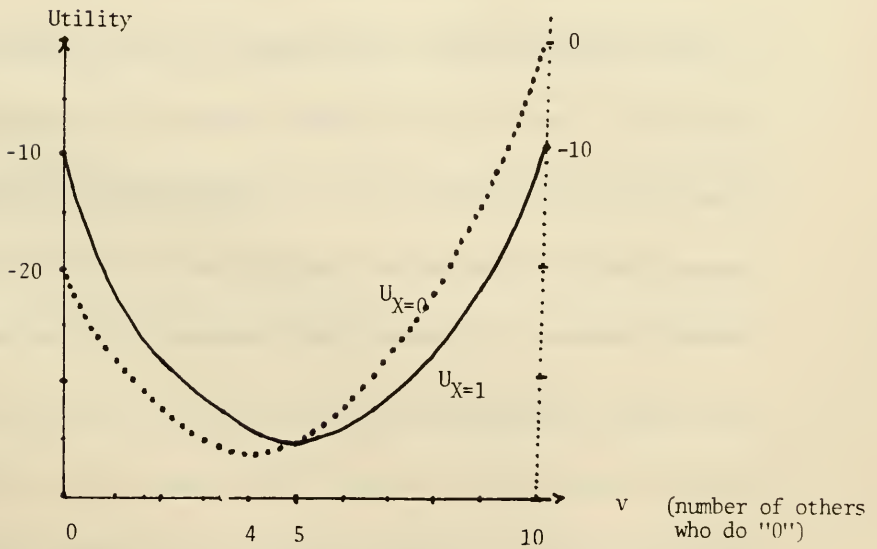


Figure 8

dilemma, where the social norm is likely to be "evade," unless appropriate institutions exist to forestall it (for instance, religious and moral values, social stigma, IRS audits and jail). The incentive to evade arises from the fear that if others do it, whoever doesn't will be exploited. A great many traps of this sort exist in society--even selling stocks in a tumbling market, which is only partly a price-based phenomenon.

#### B. Saving for the Future

Sen (1967) and Marglin (1963) made a case for the inoptimality of market saving. Let the act of saving one more unit for future generations' benefit be  $X$  or  $Y = 0$ ; let  $X$  or  $Y = 1$  be not doing this (i.e. causing an external cost). Sen argues that "Given the action of all others, each individual is better off not doing the additional unit of saving himself. Hence nobody will, (even though) everyone would have preferred one more unit of saving by each than by none." Such a world will look like the  $U_{X=1}$  and  $U_{X=0}$  curves in Figure 7, when there are  $N_1$  people in society. The dominant strategy for an individual is  $X = 1$ . If everyone in society has  $U_{X=0}$  and  $U_{X=1}$  curves such as in Fig. 7 then nobody will save the



extra unit. The result is strictly Pareto-inferior to a situation where everybody saves the unit; individual rationality leaves society at point M, rather than at S. Sen calls this the isolation paradox. Its 2x2 equivalent is prisoner's dilemma (interaction types #3 for all).

Now, let there be  $N_2$  people in society. Figure 7 is now consistent with the following description of preferences, which differs in a slight but important way from Sen's statement above for  $N = N_1$ : Given the action of all others, each individual is better off not doing the additional unit of saving himself except when (nearly) everybody else does it, in which case the individual is best off doing it, too. The latter is Vickrey's assurance paradox. It is so called because in the isolation paradox, "enforcement" is necessary for collective rationality; in the assurance paradox, "assurance" or unanimous belief in others' good will is sufficient. Use of [2] <sup>in the</sup> shows that ~~an~~ example of the "interdependence of the transfers of different donors," there is a critical mass of people,  $v_c$ , such that if  $v_c$  plus epsilon choose to save, others soon join. If  $v_c$  minus epsilon choose to save, everyone soon defects. The 2x2 game equivalent of the  $N \times 2$  assurance paradox is given by:

OTHERS

	not	
	save	save

not save	2, 2	3, 1
----------	------	------

INDIVIDUAL

[4 = best; 1 = worst]

save	1, 3	4, 4
------	------	------

This game results when players have preferences consistent with interaction

type #1. There are two Nash equilibria: 4,4 and 2,2. If no one saves

no one has an incentive to change, given the behavior of everyone else.

If everyone saves, again, no one has any incentive to change. The problem

is: how to get over the "hump" of  $v_c$ .

The apparent clash, then between the strict market failure of the isolation paradox, and the conditional market success of the assurance paradox, rests either upon the assumption of slight but important differences in preferences, or simply on how many other people there are, i.e. on the size of N.

C. When in Rome.....

The following is a true story. The arrival of the American Marines in Beirut in summer 1982, in connection with evacuation of PLO forces,

had far-reaching implications. Not the least of these were the long lines of furious Lebanese motorists trailing behind Marine vehicles at the Barbir intersection. Marine drivers made the error of stopping at red lights. Lebanese drivers, unaccustomed to such illogical and ill-considered behavior, left their cars to learn exactly what caused the American trucks to stop. Chaos resulted. This incident is helpful in building an application of (12) where some of the "a" parameters are nonlinear in v.

Suppose we arrive in a country where some people stop for red lights and others don't. Let "stop" be X and Y = 0; "don't stop," X and Y = 1.

Let a newcomer's utility function be:

$$(15) \quad U = (10 - 2v)X + (v^2 - 8v - 20)$$

where  $a(v) = 10 - 2v$  and  $b(v) = v^2 - 8v - 20$ . What should the new arrival do?

Clearly, the individual prefers that everyone (including himself) stop at red lights, since  $U_{X=0, v=N}$  exceeds  $U_{X=1, v=N}$ ,  $N = 10$  (see Figure 8). If there are 10 other people at an intersection, and he thinks over half tend to stop, it pays the individual to stop for a red light, too (see Fig. 8). If, on the other hand, no one else stops for a red light, the individual should run the light, too, since doing otherwise runs the risk

of a rear-end collision;  $U_{X=1,v=0}$  exceeds  $U_{X=0,v=0}$ . In each case, it pays the individual to do what the majority does. When in Rome, or in Beirut....<sup>13</sup>

The worst of all possible worlds for the newcomer is when half of all others stop for lights and half do not--providing, of course, we have no way of knowing who belongs to which group. In this case, no matter what a person does, chaos is at its maximum, and utility, its minimum. Such societies are unstable; sooner or later, a social convention develops where people choose uniformly one behavior or another. Schellings's (1978) example is daylight saving vs. standard time. Some countries adopt one, some another, but no country adopts DST for some people and ST for others. Conformity may be stifling, but it is also soothing for the nerves.<sup>14</sup>

### Pushing out frontiers

The decision of an individual to leave the city and join an outlying settlement depends crucially on that person's perception about how many other people will do the same. This suggests that settlement can be usefully modelled as an  $N \times 2$  game, where  $N$  potential settlers must each choose one of two strategies: remain in the city, or pioneer, and where no individual is certain what other potential settlers will choose to do .

Let a typical individual's utility function be:

$$U = (-b + av)X + cv$$

where  $b$  is the once-and-for-all costs of moving (including psychic ones),  $a$  is the utility gained from each other person who joins the settlement,  $v$  is the number of other persons who choose to settle,  $c$  is the indirect utility the individual gets from each other person who settles, irregardless of whether that person himself does, through economic development, border security, etc., and  $X=1$  is 'pioneer',  $X=0$  is "stay home".

For a person who chooses to stay in the city,  $U_{X=0} = cv$  ; for a settler ( $X=1$ ),  $U_{X=1}$  is  $-b + (a+c)v$  . Suppose everyone in the group of  $N$  potential settlers has the same utility parameters. Then Figure will serve to determine the nature and existence of equilibrium.

There are three possible scenarios.

(a) The number of potential settlers is small, equal

TYPICAL  
PERSON'S  
UTILITY

STAY AT HOME  
PIONEER

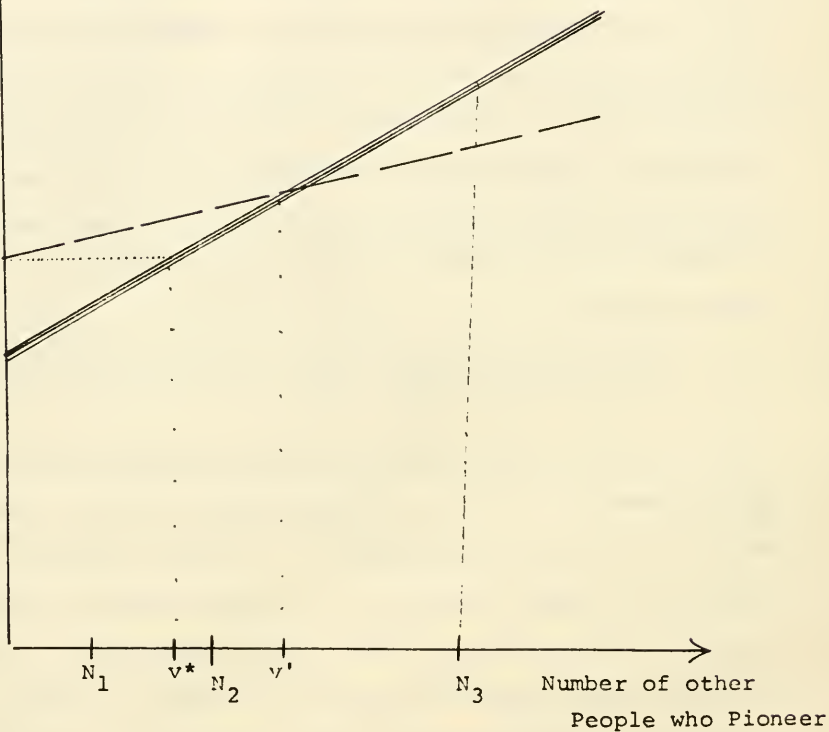


Figure. 9 . Nx2 Settlement Game

to  $N_1$  . The dominant strategy here for everyone is to stay home. In the resulting equilibrium, settlements that are established are abandoned; this is an efficient outcome. In this case:  $N_1 < b/(a+c)$

b) Suppose population growth occurs. The number of potential settlers is now  $N_2$  , where:

$$b/a > N_2 > b/(a+c)$$

Now, it is Pareto-efficient for  $N_2$  persons to pioneer. Left to choose freely, however, they will not. Each person reasons: I am best off if I remain in the city, while others go out to pioneer. When everyone thinks this way ( $X=0$  is the dominant strategy), no-one pioneers; all are worse off. This is the free-rider case, or Prisoner's Dilemma.

One way to smash this Pareto-inferior equilibrium is to enlist a coalition of persons who pledge to pioneer together. The smallest such coalition, such that it pays to join rather than have everyone stay home, is:  $v^*$  . Note, however, that this coalition is unstable (free rider games have no core). For someone who has joined a settlement of  $n^*$  or more people, it pays to leave and return to the city, unilaterally (provided no one else follows). When everyone reneges in this way, the settlement collapses, a not uncommon phenomenon.

(c) After more population growth, the number of potential settlers grows to  $N_3$  where

$$N_3 > b/a$$

This is a "critical mass" externalities game, type 1. There is no dominant strategy. An individual's decision to pioneer or stay home depends crucially on whether he or she thinks more than  $v'$ , or fewer than  $v'$ , people will join.  $v'$  is the 'critical mass'. Suppose everyone believes that fewer than  $v'$  people will join. Then the best strategy is  $X=0$ ; everyone stays home. This is one possible Nash equilibrium. There is a second Nash equilibrium. This occurs when people each believe that more than  $v'$  people will choose to pioneer. Then the best strategy is  $X=1$ ; soon, all  $N_3$  potential settlers have joined. The resulting equilibrium is as efficient, and as stable, as the stay-at-home one. In this case, nothing more is needed to generate a successful frontier settlement program than to persuade a critical mass of  $v'$  people to join. Once this is done, the program generates its own momentum.



## Conclusion

The fundamental theorem of both micro- and macro-economics is still the efficiency with which competitive markets allocate resources. Market distortions owing to external costs and benefits have been modelled and debated to a large extent as a technical problem -- improper relations among goods and resources. But externalities are in essence a human problem, arising from interactions among people. Seen in this way, the problem of externalities can best be modelled in a game-theoretic framework. Such a framework should find applications beyond the relatively narrow pollution-noise-crowding concerns of economics.

## FOOTNOTES

1. See S. Maital, *MINDS, MARKETS & MONEY*, Basic Books: New York, 1982; and George C. Lodge, *The American Disease*, Knopf, New York, 1984.

2. Becker (1981) has pioneered this type of analysis.

3. Davis & Whinston (1962) were among the first to realize this, and their paper is still widely cited even after more than two decades.

4. See also Atkinson & Stiglitz (1980).

5. In market economies, all individuals are highly interdependent even when there are no externalities, because, as Mishan (1971) pointed out, "an exogenous change in the behavior of individuals can alter the equilibrium set of product and factor prices and thereby alter the utility level of other persons" (p. 2). This price-based interdependence generates efficiency rather than disturbing it. In Hyman's (1983) example, if my hobby is photography, increased demand for photographic equipment, chemicals and paper by other hobbyists makes me worse off by making my avocation more expensive. This is not a true externality, because efficiency requires that scarcer goods (higher demand relative to supply) become more costly.

Sometimes, the distinction between price effect and externality is blurred. When people demand higher wages, or hike their prices, or accelerate their spending, in anticipation of general price rises (inflation), is that behavior an external cost to others who do not act likewise? Or is it simply the working of an efficient market? See Maital and Benjamini (1980), and Sutcliffe (1982).

6. Ingenious attempts to invent missing markets have been made. Some markets turn out not to be missing at all. One of the earliest examples of externalities was Meade's 1952 apple orchards that provide food for bees, and bees that pollinate apple trees. In choosing the size, location and number of their hives, beekeepers presumably ignore the external benefits of their bees to orchards; and in selecting the size, nature and location of orchards, growers presumably ignore the benefits of their trees to honey producers. However, Cheung (1973) showed that a market does exist in pollinating services; fruit growers can and do rent bees. The rent presumably reflects the honey rental-bees' produce.

Admittedly, the shift from partial equilibrium to general equilibrium models in externality theory, pioneered by Ayres and Kneese (1969) was a major step forward. But characterizing a non-existent market (e.g. current consumption 'feasts' to which future generations are not invited) is often very difficult.

7. "The general equilibrium theory was a masterful first step in seeing the economic problem clearly in a non-institutional or pre-institutional non-biological static context. The non-cooperative solution concept provides the means for the analysis of richer and more relevant models for political economy and other social sciences." Shubik, 1982, p. 385.

8. The interactions described below among people are generated by two different types of processes: objective technological relations, which translate actions (i.e. labor supply) into goods and services, as well as 'bads', and subjective psychological relations, which translate goods and bads into utility. A complete model of externalities would try to separate out these underlying forces and characterize them completely. The point made here is that a large literature on such aspects of externalities exists, but a relatively small one on interpersonal interactions and their implications.

In this paper,  $U=F(X,Y)$  is a kind of reduced-form specification not unlike indirect utility functions where the arguments are prices and incomes. The function  $F(\ )$  blends both technology and psychology. The behavioral and engineering constraints underlying utility functions can be illustrated in at least three different ways. *oblique*

I. Players A and B each can produce a distinct good, R and S, respectively. When, and only when, both R and S are produced, they interact to produce a third good, Z. The technology is such that:  $R = rX$ ,  $S = sY$ , and  $Z = zX \cdot Y$ ,  $X, Y = 0, 1$ . Each player's utility is proportional to the sum of the three goods produced, with signs reflecting whether the good adds to or detracts from utility:

$$U = a(R+S+Z) = a_1X + a_2Y + a_3X \cdot Y, \quad a_1=ar, \quad a_2=as, \quad a_3=az$$

$$V = b(R+S+Z) = b_1X + b_2Y + b_3X \cdot Y, \quad b_1=br, \quad b_2=bs, \quad b_3=bz$$

For instance, let A be an electric power plant discharging thermal waste (R) and B be a nearby firm discharging organic waste (S). Thermal waste and organic waste together -- warm water plus sludge -- create Z, algae. The root of this nonlinear interaction is technological.

II. Two goods R and S are jointly produced by the actions of players A and B. The technology is nonlinear:  $R=r_1X + r_2Y + r_3X \cdot Y$ ,  $S=s_1X + s_2Y + s_3X \cdot Y$ . Player A's utility depends only on good R; player B's, only on good S.

$$U = a(R) = ar_1X + ar_2Y + ar_3X \cdot Y = a_1X + a_2Y + a_3XY$$

$$V = b(S) = bs_1X + bs_2Y + bs_3X \cdot Y = b_1X + b_2Y + b_3XY$$

Let A be a group of commuters using only an east-west highway, and B, commuters using only a north-south highway. Let the highways intersect, with no traffic light existing. R, then, is east-west travel; S, north-south. When A-commuters head home first, a line of A-cars blocks the intersection for B-cars; similarly, when B-commuters are first. When A and B both try to cross the intersection at the same time, chaos results. (The resulting

game is one of pre-emption; quickest on the draw wins).

III. Four different goods are produced. Player A produces goods R (from which he benefits) and byproduct T, from which B suffers. Similarly, B produces goods S (from which he benefits) and W, from which A suffers.

If the technology is linear,  $R=r_1X$ ,  $T=t_1X$ ,  $S=s_1Y$ ,  $W=w_1Y$ . If utilities are proportional to the sum of the relevant goods:

$$U = a(R+W) = ar_1X + aw_1Y = a_1X + a_2Y$$

$$V = b(S+T) = bs_1X + bt_1Y = b_1X + b_2Y$$

This is externalities model (8). Now, if either technologies are nonlinear, e.g.  $R=r_1X + r_2XY$ , etc., or utilities are nonlinear, i.e. own-goods and opponent byproducts interact, so that  $U=f(R, W, R \cdot W)$ , we get externalities model (9).

For example, consider two groups, joggers and cyclists, in Central Park. Park paths give joggers pleasure, but joggers get in the way of cyclists. Cyclists enjoy biking on the paths, but endanger joggers. Joggers and cyclists interact either technologically (congestion), or psychologically (cyclists and joggers both worry when the two groups are intermingled; even though there may be plenty of room for everyone).

For purposes of this paper, it is immaterial whether models I, II or III hold true; what matters is that U and V can reasonably be assumed to be nonlinear in X and Y.

9. See Baumol (1952), Schotter (1980), and Maital & Maital (1984).

10. Peter Blau, "Interactions: Social Exchange", in David Sills, ed., International Encyclopedia of the Social Sciences.

11. See Calabresi & Melamed, "Property Rules, Liability Rules and Inalienability", Harvard Law Review, 1089-1093, 1115-1124, 1972.

12. For discussions of the pervasiveness of this two-equilibrium game, see Dybvig & Spatt (1983), Diamond & Dybvig (1984), and Liebermann & Syrquin (1984). Diamond & Dybvig portray bank runs as such a situation, and note that "depositors are insured against losing money in bank runs, thereby removing an original cause (of them)". Dybvig and Spatt treat adoption externalities (e.g., adopting the metric system, or some other standard) in a similar way; here, the government intervenes to nudge society from an inferior equilibrium to a better one, and also acts to change the nature of the 'better' equilibrium, through taxation. Liebermann & Syrquin, dealing with interactions involving transfer of property rights, note that "once a critical minority is observed to violate the principle of abiding by social norms or by formal law, a rapid erosion of the principle will probably follow. This erosion amounts to a real depletion of the social capital stock."

13. Or, Tel Aviv. As a new immigrant to Israel in 1967, unfamiliar with the local "1" 's and "0" 's, I made the tragic error of braking when a traffic light changed from green to amber. The Austin coupe that demolished the rear of my Peugeot cured that mistake for all time.

14. See (Jacobsen 1982) for an interesting study of what happens when different ethnic groups living in a single country hold different norms.

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